On the Matter of Aggregate Models for Syllogistic Reasoning: A Transitive Set-Based Account for Predicting the Population

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Syllogistic Reasoning

All scientists are gods
Some gods are immortal

What, if anything, follows?

- Reasoning is a core skill of human cognition
- Core domain: syllogisms, i.e., categorical quantified assertions

Example: Atmosphere Heuristic

All scientists are gods

Some gods are immortal

Logic: No Valid Conclusion

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Data: Some scientists are immortal (**70%**)

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Example: Atmosphere Heuristic

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- Theories try to capture the deviations from logic
- Example: Atmosphere heuristic[1] predicts quantifier
  - by merging quantity and polarity
  - . . . but no statement about the direction

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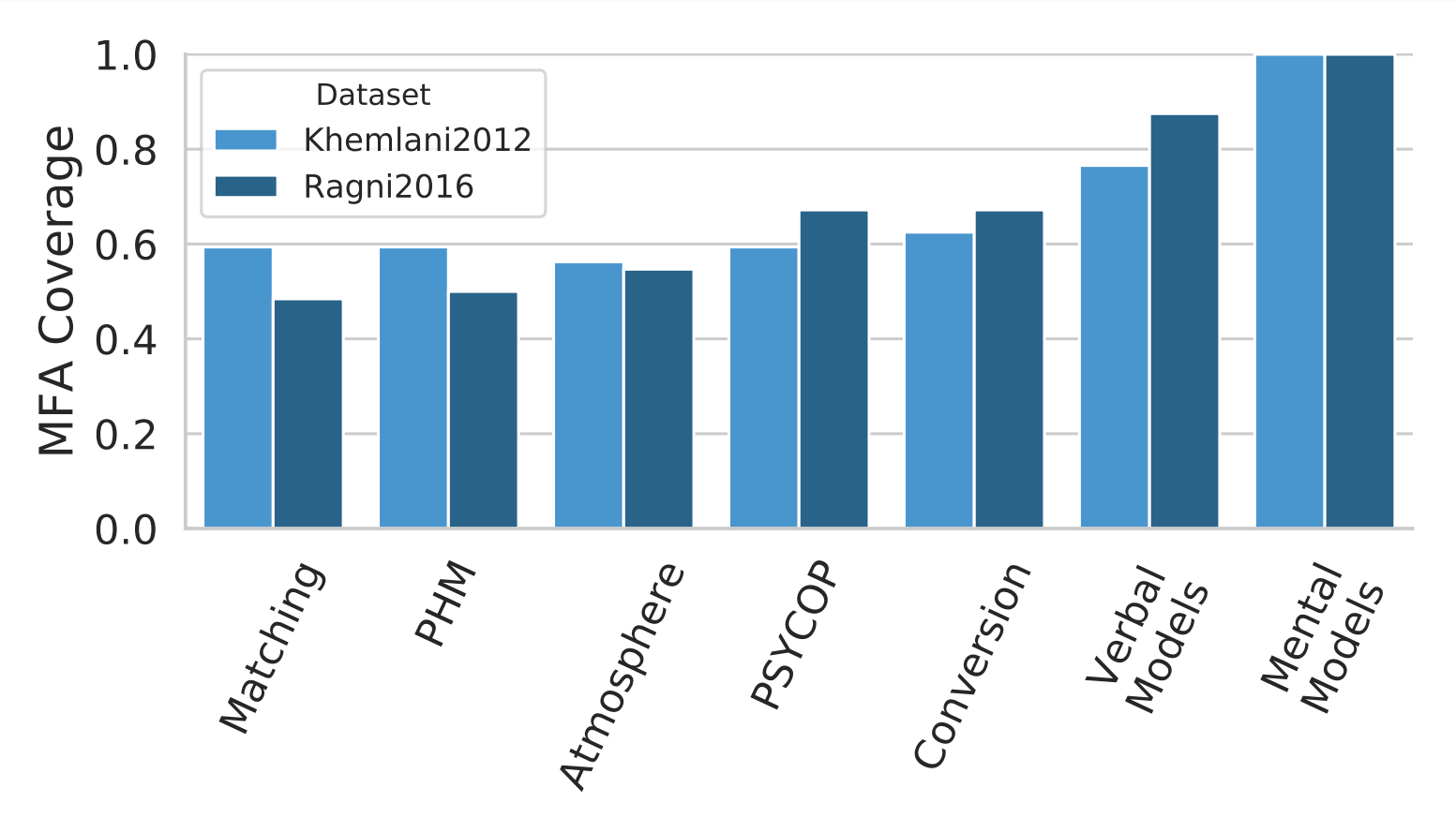
Theories of Syllogistic Reasoning (Khemlani & Johnson-Laird, 2012)

Heuristics	Formal Rules	Diagrams, Sets & Models
Atmosphere	PSYCOP	Euler Circles
Matching	Verbal Substitutions	Venn Diagrams
Illicit Conversion	Source-Founding	Verbal Models
Probability Heuristics	Monotonicity	Mental Models

- Meta-analysis demonstrates: no single best performing theory
- Heuristic approaches perform worse than model-based approaches

Covering the Most Frequently Given Answer



Research Question

Are simple heuristic strategies simply *insufficient* for predicting human syllogistic reasoning?

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Can we identify *simple mechanisms* that explain inferences?

Heuristic Principles

- We need to identify fundamental principles of heuristics
- Requirements for good heuristics, they
  - Should work in many practical situations
    (logically valid when applied correctly)
  - Should not require deep reasoning process (akin to pattern matching)
  - Should leave room for illogical inferences
    (application in unwarranted cases)

Transitivity

Transitivity is a core principle and good heuristic:

1. Works in practice:
    - Basic principle for making inferences

[2] Goodwin, G. P., & Johnson-Laird, P. N. (2008). Transitive and pseudo-transitive inferences. *Cognition, 108(2)*, 320-352

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2. Pattern matching:
    - Attempts to find simple paths of information flow (A-B-C)
    - Conclusion is intuitive

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2. Pattern matching:
    - Attempts to find simple paths of information flow (A-B-C)
    - Conclusion is intuitive
3. Room for illogical inferences:
    - Transitivity is often applied in unjustified cases (pseudo-transitivity)[2]
    - Participants might force a task into a transitive shape

[2] Goodwin, G. P., & Johnson-Laird, P. N. (2008). Transitive and pseudo-transitive inferences. *Cognition, 108(2)*, 320-352

Syllogistic Domain

- Total of 64 problems consisting of
  - 4 quantifiers (All, Some, Some ... not, None)
  - 4 figures depending on arrangement of terms (A, B, C)

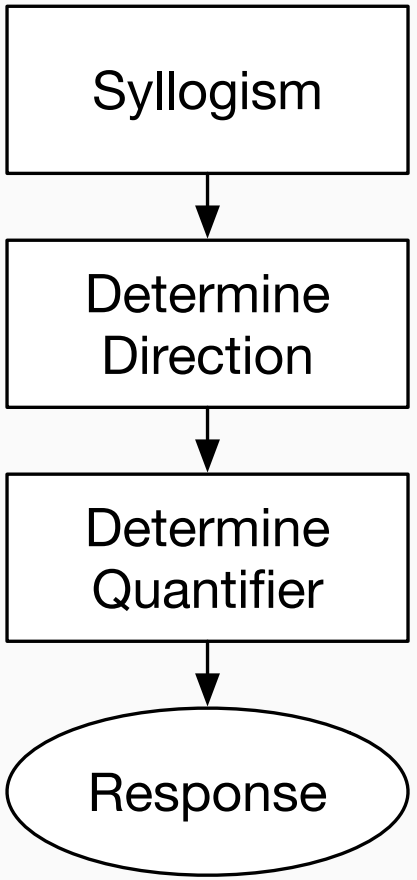
Figure 1	Figure 2	Figure 3	Figure 4
A-B	B-A	A-B	B-A
B-C	C-B	C-B	B-C

- Nine possible conclusions:

  Eight conclusions relating end terms (A, C) and "No Valid Conclusion" (NVC)

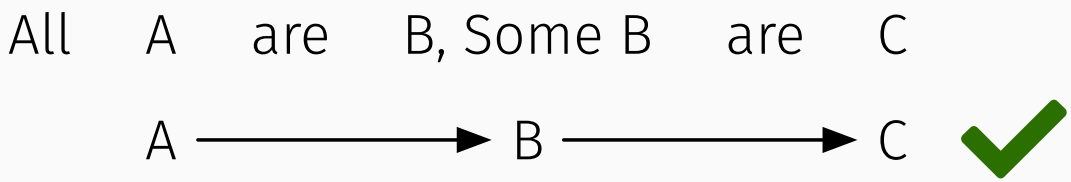
The TransSet Model

1. Determine direction
   Search for a transitive path and determine the direction of the conclusion
2. Determine quantifier
   Propagate a set along the path



Determine Direction: Finding a Transitive Path

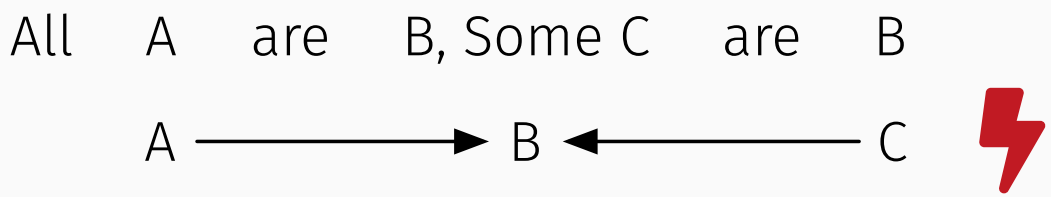
Figure 1:



- Transitive path directly available (A-B-C)
- Analogously possible for Figure 2 (C-B-A)
- Directly yields A-C (Figure 1) and C-A direction (Figure 2)

Determine Direction: Finding a Transitive Path

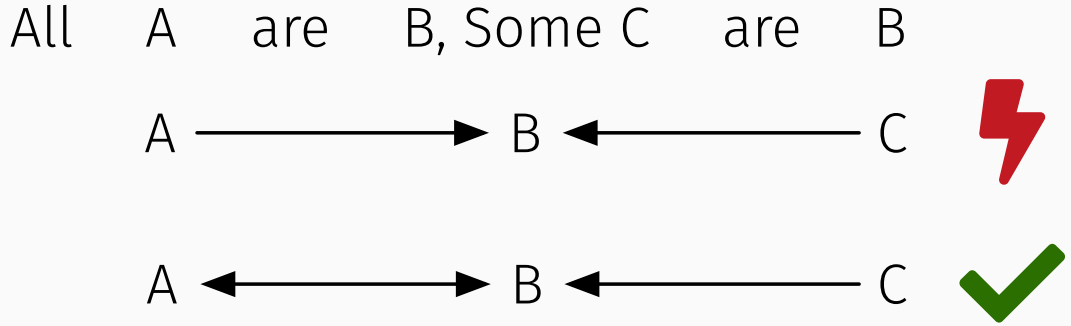
Figure 3:



- No direct path available
- Assumption: Reasoners change task structure to enforce a path
- NVC if path cannot be found

Determine Direction: Finding a Transitive Path

Figure 3:

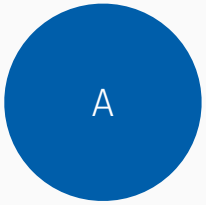


- Premises with universal quantifiers (All, No) treated bidirectionally
- Yields same path structures as for Figure 1 and Figure 2 syllogisms
- Same mechanism for Figure 4 syllogisms

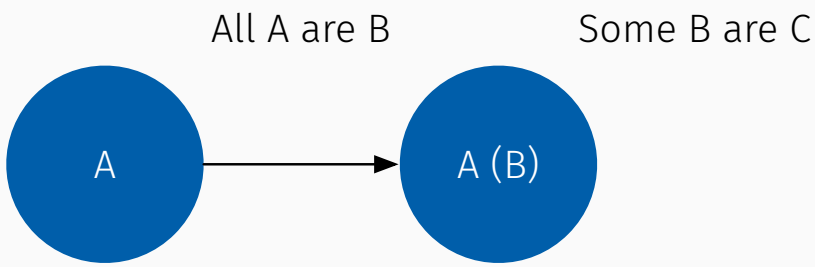
Determine Quantifier: Set Propagation

All A are B

Some B are C



Determine Quantifier: Set Propagation



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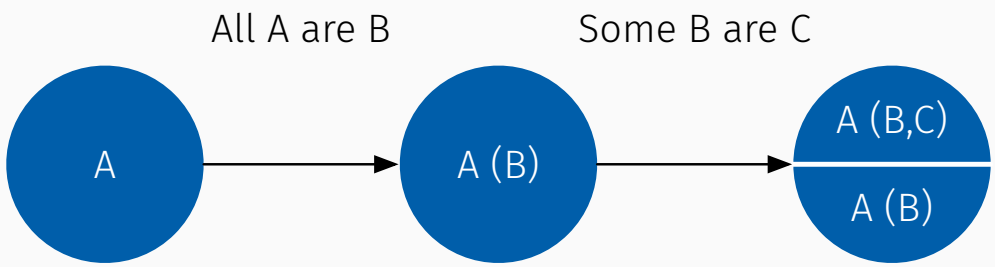

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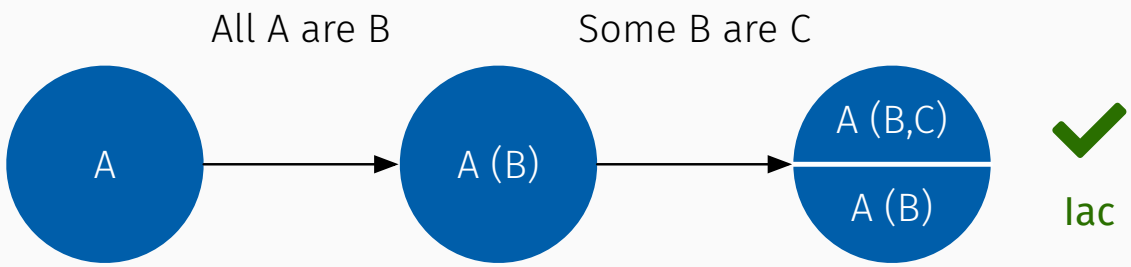
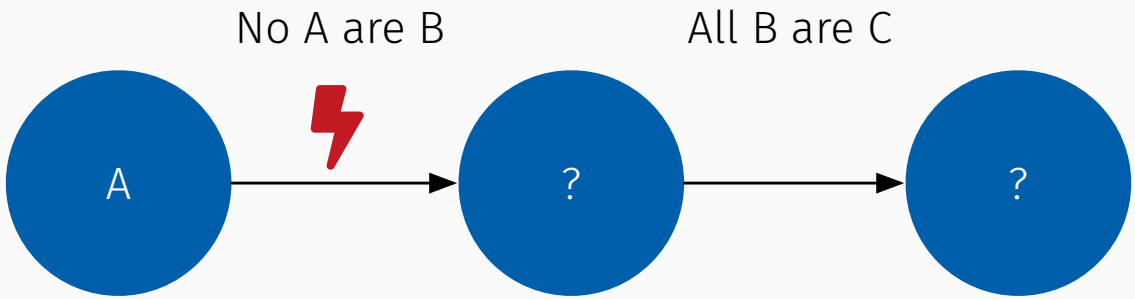
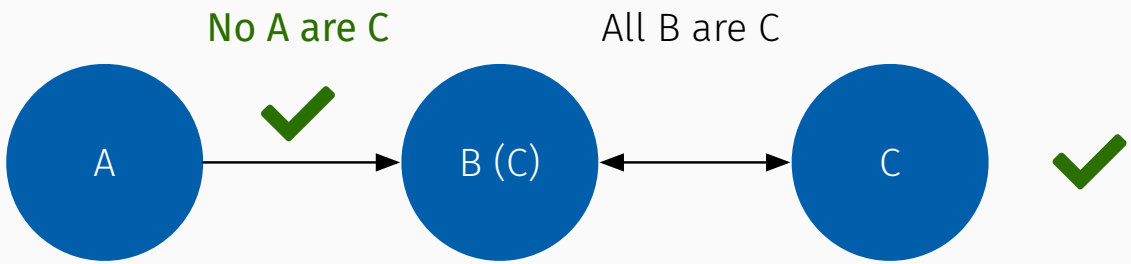


Determine Quantifier: Set Propagation - Conflict



- Ambiguity of "No" as first quantifier: Empty set vs "All A are no B"
  - Empty set: No statement about elements of A
  - "No A are B" interpreted as "All A are no B"
- Set propagation fails

Determine Quantifier: Set Propagation - Conflict Resolution



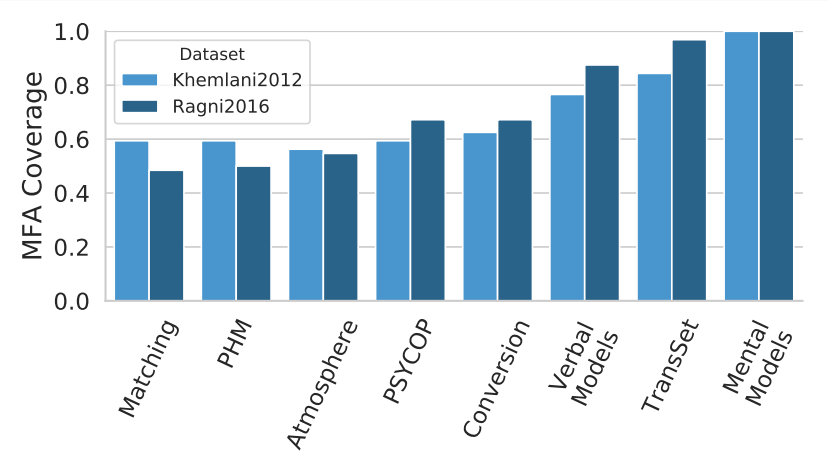
- Start from the end of the path
- Bidirectional interpretation if second premise quantifier is “All”
- Simplifies ambiguity and leads directly to the conclusion

Analysis

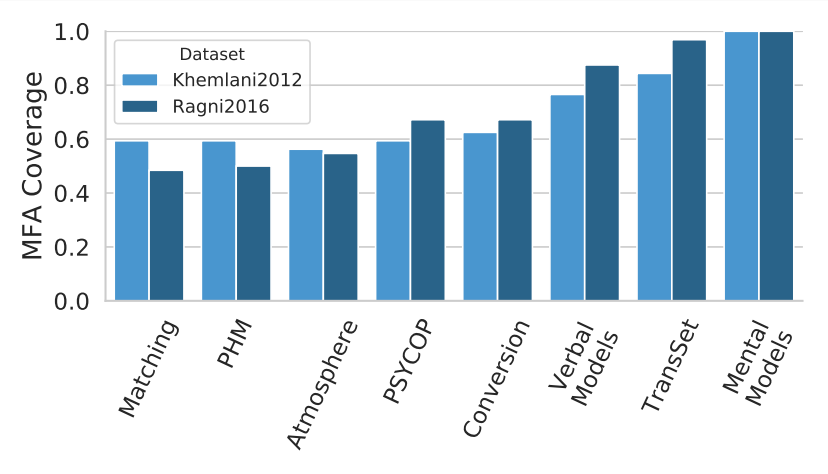
Comparison of models with most-frequent answer (MFA)

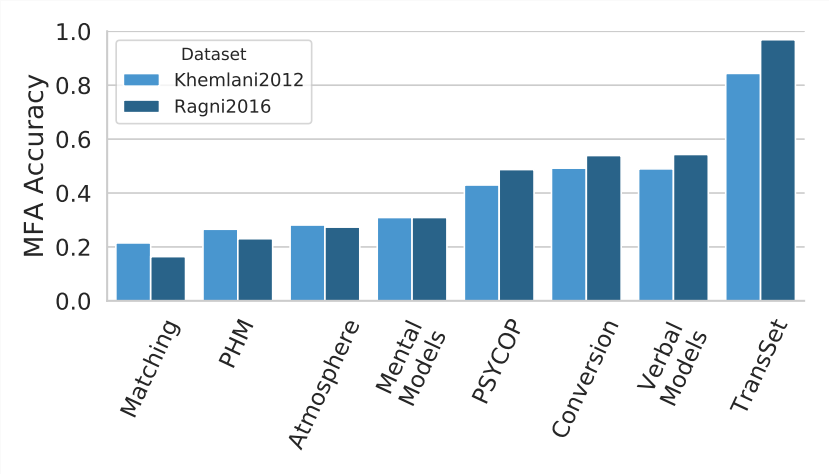
- MFA is the optimal response strategy for aggregate prediction models
- Coverage: Check if MFA is in set of possible model predictions
- Accuracy: Discount coverage score based on number of possible predictions

MFA Analysis



MFA Analysis





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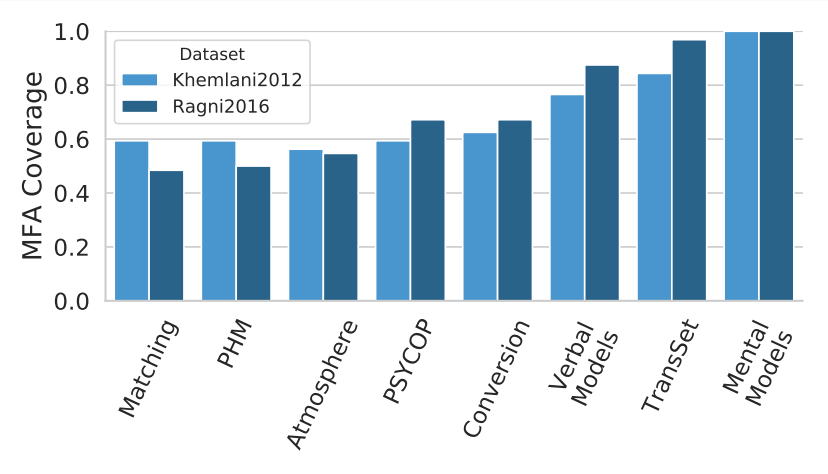
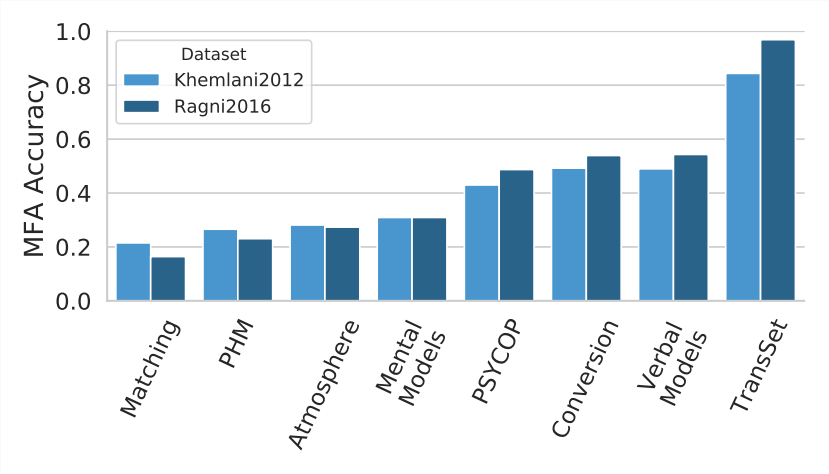
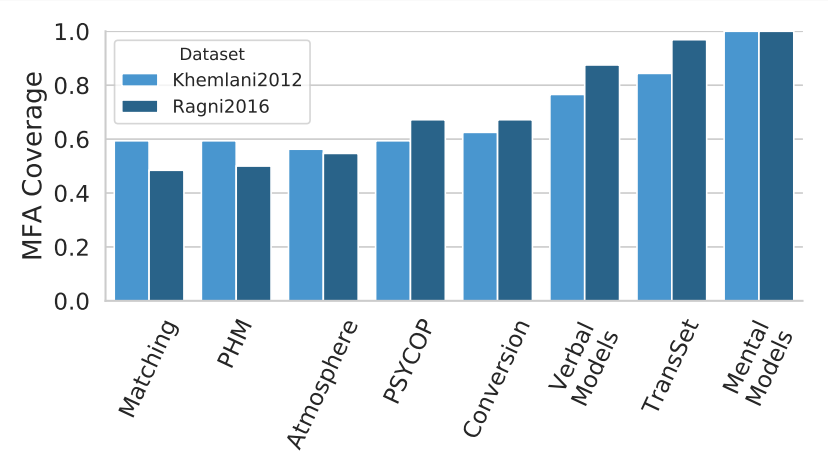
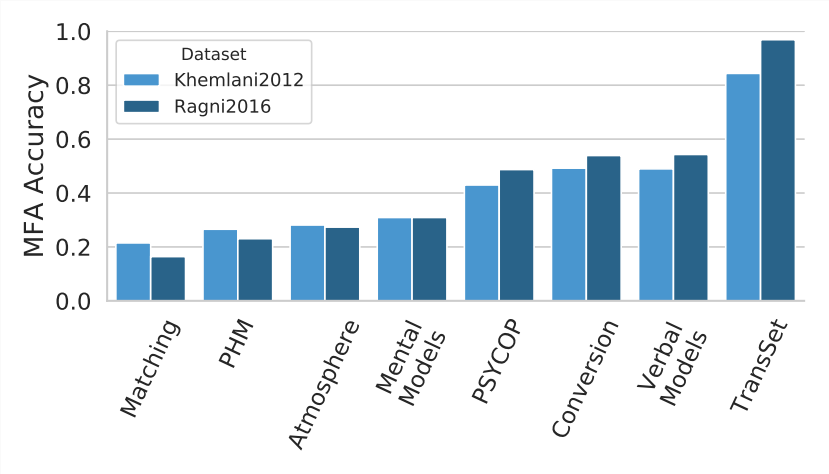




- TransSet achieves peak performance

MFA Analysis



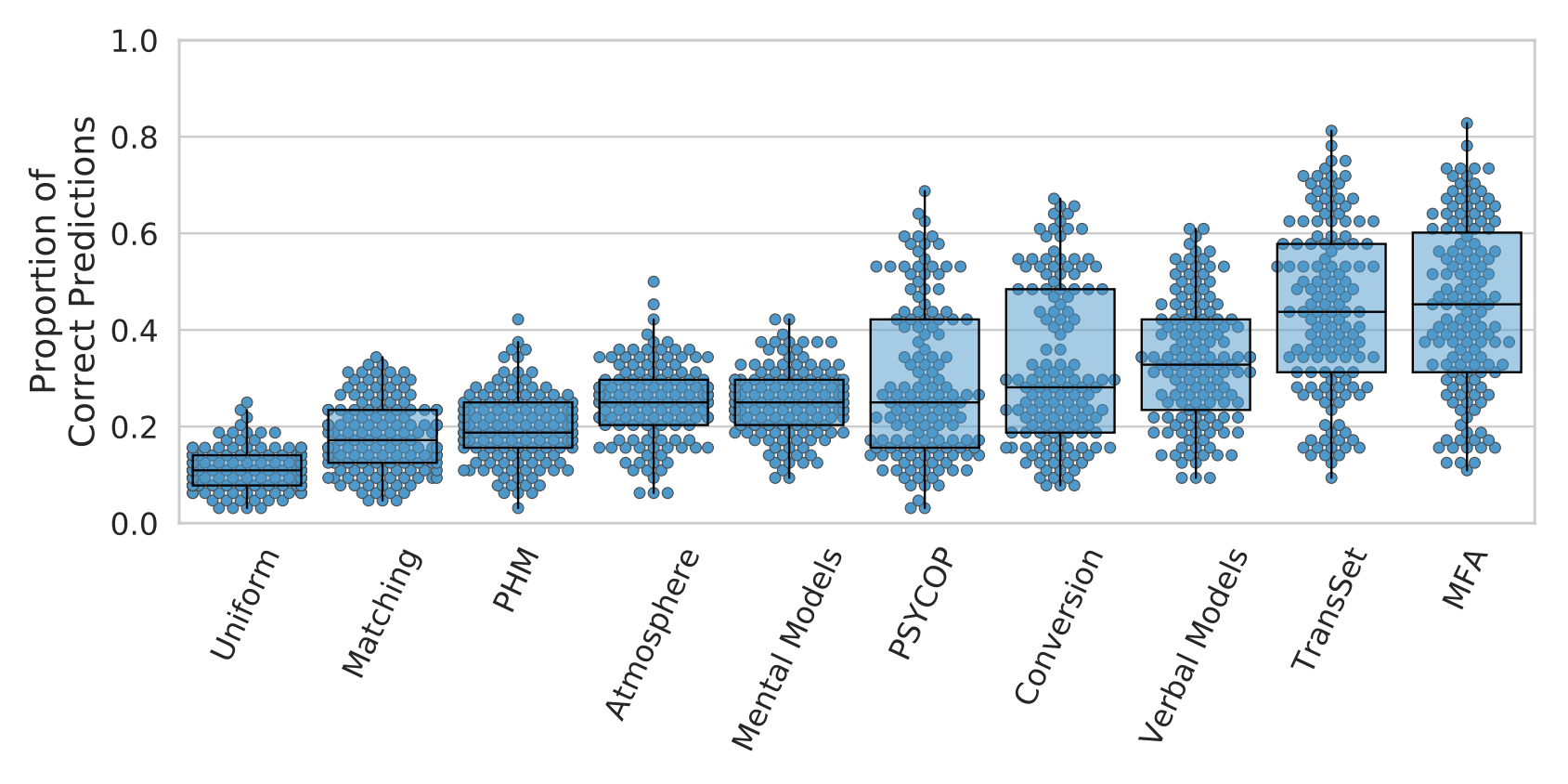


- TransSet achieves peak performance
- Cognitive models drop in performance when penalized for multiple responses
  - Highlights unspecificity of model predictions
  - Suggests severe shortcomings of the predictive forms of the models

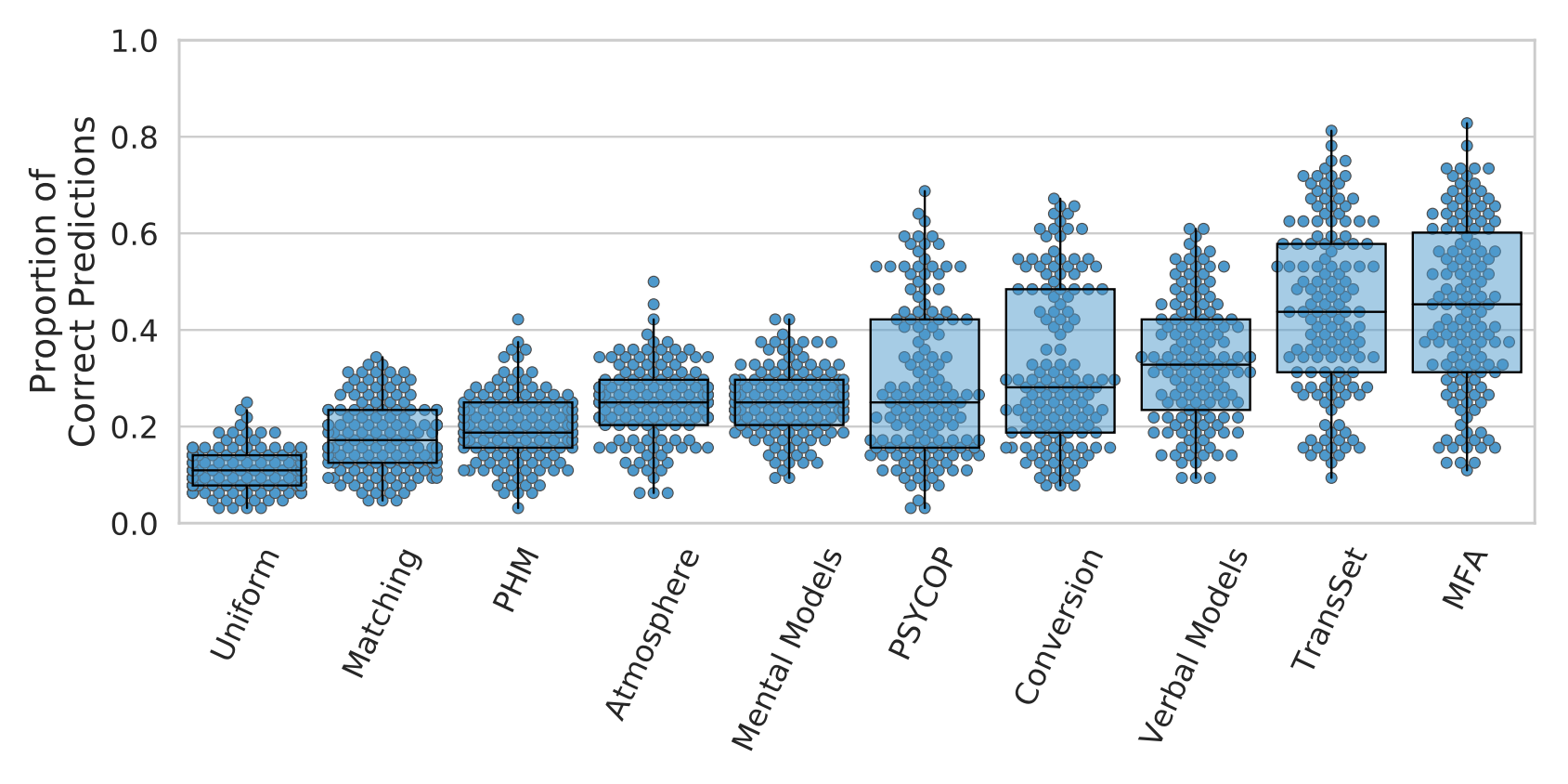
Individualized Analysis

- Investigate how applicable reasoning strategies are to individual reasoners
- For individuals, evaluate the predictive accuracy on their responses (proportion of correct predictions)
- Heuristic models should be able to accurately predict a small number of participants and perform rather poorly on the rest

Individualized Analysis



Individualized Analysis



- Complex models are unsuitable, unless they can fine-tune predictions
- Large variance of MFA predictions
  - Highlights the limit of aggregation-based strategies
  - "Average reasoner" is an unsuitable representation for an individual

Conclusion

- TransSet is able to capture human reasoning data fairly well while adhering to known statistical effects and psychological phenomena:
  - Figural effect (Johnson-Laird, 1983)
  - Conversion (Chapman & Chapman, 1959)
  - Informativeness of quantifiers (Chater & Oaksford, 1999)
- Occam's Razor: questions worth of complex fit-based models
  - Unnecessary for modeling syllogistic reasoning unless able to be fine-tuned to individuals
  - TransSet as a simple heuristic suffices for population-based aggregate predictions

Thank You!

References

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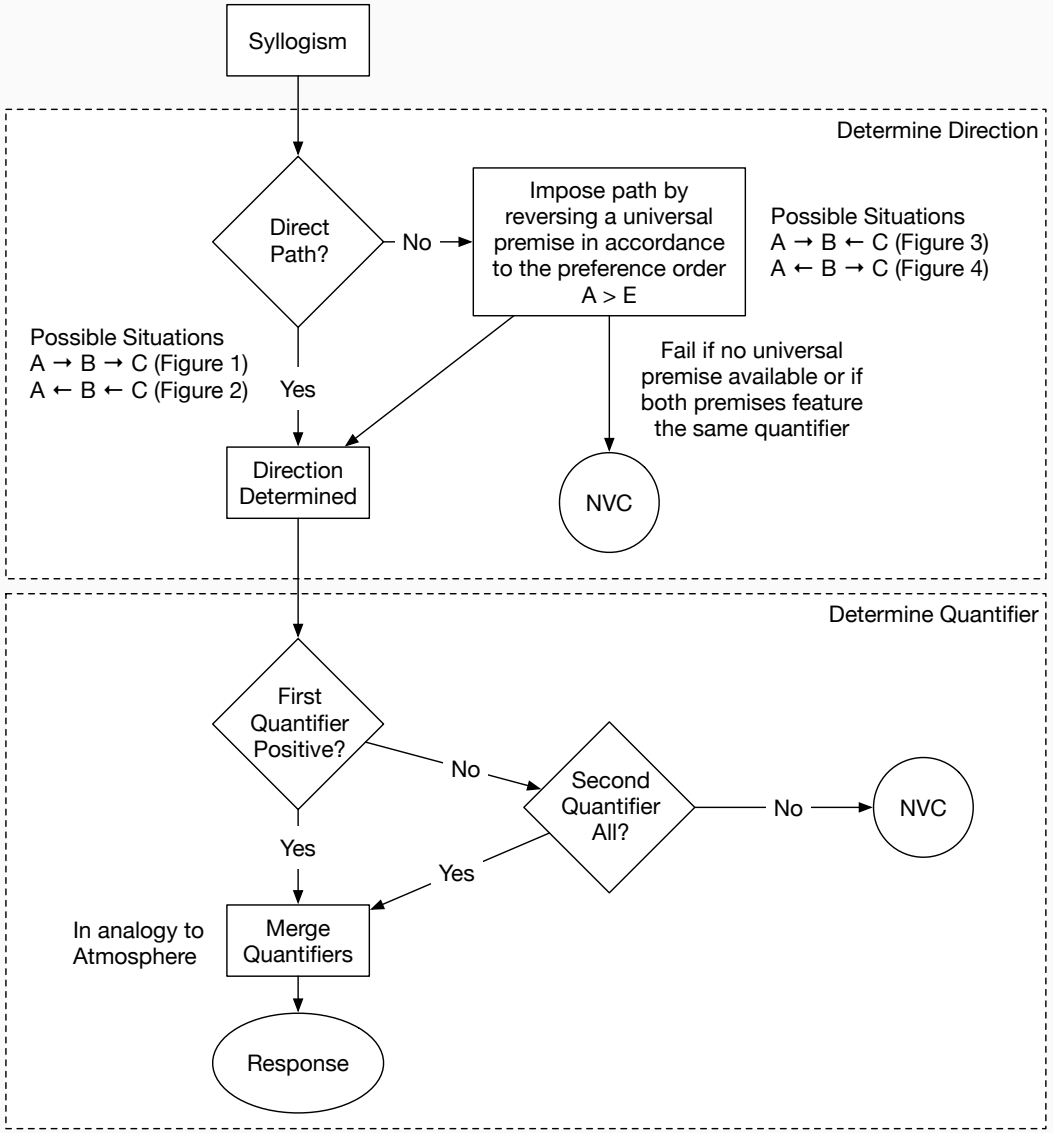
Code on GitHub:

`https://github.com/Shadownox/iccm-transset`

Model Flow


Syllogism
Determine Direction
Direct Path?
No
Impose path by reversing a universal premise in accordance to the preference order A > E
Possible Situations
A → B ← C (Figure 3)
A ← B → C (Figure 4)
Possible Situations
A → B → C (Figure 1)
A ← B ← C (Figure 2)
Yes
Fail if no universal premise available or if both premises feature the same quantifier
Direction Determined
NVC
Determine Quantifier
First Quantifier Positive?
No
Second Quantifier All?
No
NVC
Yes
Yes
In analogy to Atmosphere
Merge Quantifiers
Response